MIXED-INTEGER DISCIPLINED CONVEX PROGRAMMING

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“Models with integer and binary variables must still obey all of the same disciplined convex programming rules that CVX enforces for continuous models. For this reason, we are calling these models ***mixed-integer disciplined convex programs*** *or* ***MIDCPs****.“*

First, **mixed-integer linear programming (MILP)**,

$$\begin{aligned} \min_x \; & c^T x \\ \text{subject to } & Ax = b, \\ & x \geq 0, \\ & x_i \in \mathbb{Z}, \forall i \in I \end{aligned}$$

- Despite NP-Hardness, many problems of practical interest can be solved to optimality or near optimality
- Algorithms are based on LP relaxations, branch & bound, cuts, preprocessing, heuristics, ...
- 50+ years of commercial investment in developing these techniques

WHO USES MILP?

http://www.gurobi.com/company/example-customers

Minimize unit-production costs + fixed operating costs

subject to:

- Total generation = total demand, hourly over 24h
- Generation and transmission limits
  - If a generator is on, it produces within some interval $[l, u]$, otherwise its production is zero.
- Linear approximation of nonlinear powerflow laws

1% reduction in generation gives billions of dollars per year cost reduction. Worth spending time to improve optimality.

Mixed-integer convex programming (MICP),

$$\begin{aligned} \min_x f(x) \\ \text{subject to } g_j(x) \leq 0, \forall j \\ x_i \in \mathbb{Z}, \forall i \in I \end{aligned}$$

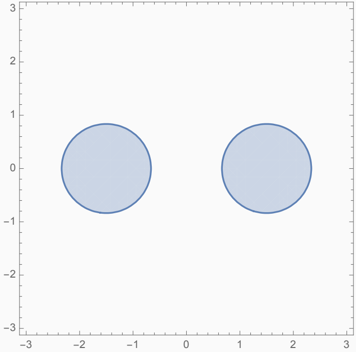
Objective function f and constraints g_j **convex**

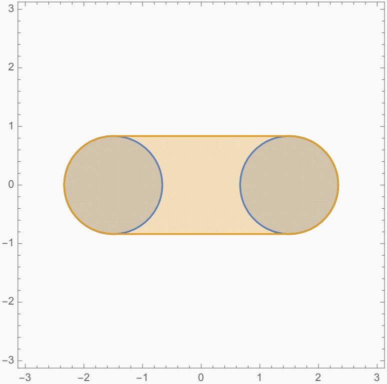
Why mixed-integer convex over MILP?

- Risk/uncertainty
- Statistical model fitting (Bertsimas and King, 2015)
- Model predictive control
- Add discrete aspects to anything you already model using CVX

Disjunctions/unions of compact convex sets (Stubbs and Mehrotra, 1999)

$$\{x : f_1(x) \leq 0 \text{ or } f_2(x) \leq 0\}$$





x belongs to the convex hull iff $\exists\, u_1, u_2, \lambda_1, \lambda_2$ such that

$$x = \lambda_1 u_1 + \lambda_2 u_2$$

$$\lambda_1 + \lambda_2 = 1$$

$$\lambda_1 \geq 0, \lambda_2 \geq 0$$

$$f_1(u_1) \leq 0$$

$$f_2(u_2) \leq 0$$

$$
\begin{aligned}
x &= \lambda_1 u_1 + \lambda_2 u_2 \\
\lambda_1 + \lambda_2 &= 1 \\
\lambda_1 \geq 0, \lambda_2 &\geq 0 \\
f_1(u_1) &\leq 0 \\
f_2(u_2) &\leq 0
\end{aligned}
$$

- If we impose $\lambda_1, \lambda_2 \in \{0, 1\}$ then have conditions for the union, but not convex

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\end{aligned}
$$

- If we impose $\lambda_1, \lambda_2 \in \{0, 1\}$ then have conditions for the union, but not convex
- Instead, introduce $z_1 = \lambda_1 u_1$, $z_2 = \lambda_2 u_2$

$$x = z_1 + z_2$$
$$\lambda_1 + \lambda_2 = 1$$
$$\lambda_1 \geq 0, \lambda_2 \geq 0$$
$$f_1(z_1/\lambda_1) \leq 0$$
$$f_2(z_2/\lambda_2) \leq 0$$

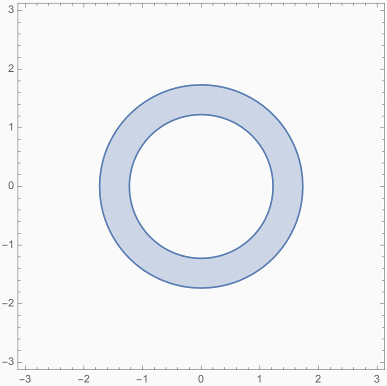
- Still not convex

$$
\begin{gathered}
x = z_1 + z_2 \\
\lambda_1 + \lambda_2 = 1 \\
\lambda_1 \geq 0, \lambda_2 \geq 0 \\
\lambda_1 f_1(z_1/\lambda_1) \leq 0 \\
\lambda_2 f_2(z_2/\lambda_2) \leq 0 \\
\lambda_1, \lambda_2 \in \{0, 1\}
\end{gathered}
$$

- *Perspective* function of any convex function is convex.

- Which nonconvex sets can be modeled with MICP? Finite unions of closed, convex sets

Conjecture: the following set is not MICP representable



State of the art for solving MICP

$$\min_x f(x)$$
$$\text{subject to } g_j(x) \leq 0, \forall j$$
$$x_i \in \mathbb{Z}, \forall i \in I$$

- f, g_j convex, **smooth**
- Oracles for gradients and Hessians are available.
- Exploit derivative-based nonlinear programming (NLP) solvers
- Bonmin, KNITRO, α-ECP, DICOPT, FilMINT, MINLP_BB, SBB, ...

> *[...] solution algorithms for [MICP] have benefit from the technological progress made in solving MILP and NLP. However, in the realm of [MICP], the progress has been far more modest, and the dimension of solvable [MICP] by current solvers is small when compared to MILPs and NLPs.*

– Bonami, Kilinç, and Linderoth (2012)

Mixed-integer disciplined convex programming (MIDCP),

$$\begin{aligned} \min_x \; & f(x) \\ \text{subject to } & g_j(x) \leq 0, \forall j \\ & x_i \in \mathbb{Z}, \forall i \in I \end{aligned}$$

Objective function f and constraints g_j **disciplined convex**

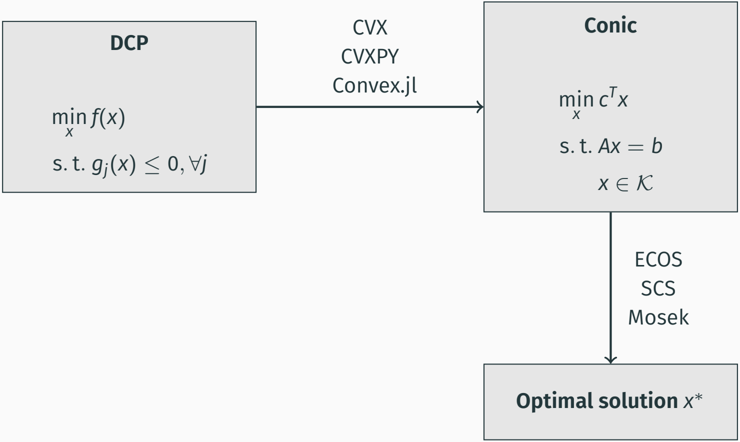
Forget about derivatives

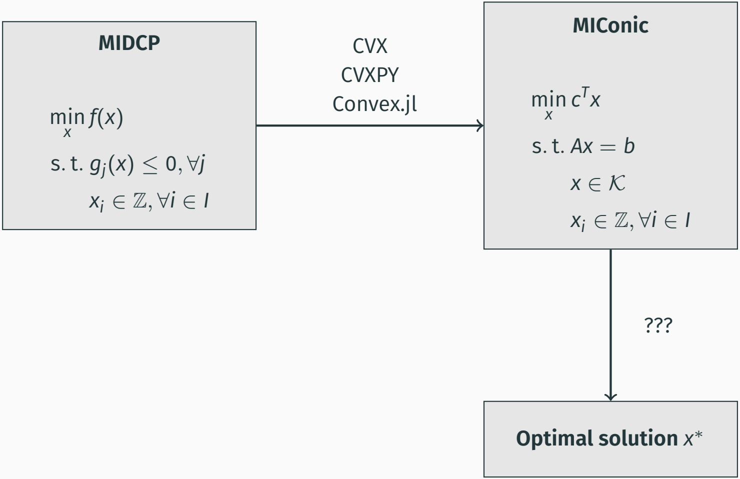
What is a disciplined convex function? (Grant, Boyd, Ye, 2006)

- Expressed as composition of basic atoms. Rules of composition prove convexity.
- Example $\sqrt{x^2 + y^2}$ versus $||[x, y]||_2$, $\sqrt{xy}$ versus geomean(x, y).

Why disciplined convex?

- Functions expressed in this form can be automatically converted to conic optimization problems. We have good solvers and theory for conic optimization.


DCP
$\min_x f(x)$
s. t. $g_j(x) \leq 0, \forall j$
CVX
CVXPY
Convex.jl
Conic
$\min_x c^T x$
s. t. $Ax = b$
$x \in \mathcal{K}$
ECOS
SCS
Mosek
Optimal solution x^*


MIDCP
$\min_x f(x)$
$\text{s.t. } g_j(x) \le 0, \forall j$
$x_i \in \mathbb{Z}, \forall i \in I$
CVX
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$\min_x c^T x$
$\text{s.t. } Ax = b$
$x \in \mathcal{K}$
$x_i \in \mathbb{Z}, \forall i \in I$
???
Optimal solution x^*

- Mixed-integer second order cone programming (MISOCP) has rapidly improving commercial support by Gurobi/CPLEX
    - Gurobi claims 2.8x improvement from version 6.0 to 6.5 (1 year)
- Otherwise (exponential cones, SDP), mainly research codes (`ecos_bb` by Han Wang, 2014; `SCIP-SDP` by Mars, 2012)
- For MISOCP, no established open-source codes

- Relax integer constraints. Solution provides valid lower bound on optimal value
- Pick a variable i to "branch on". Divide search space into $x_i \leq A$, $x_i \geq A + 1$. Solve relaxations on each of the two branches.
- If relaxation value on a branch is worse than the best known solution, then can prune that branch.

Implemented in ecos_bb

`fo9` facility layout problem

- 182 variables (72 binary), 343 constraints
- `ecos_bb`: Not solved in 11 hours, no feasible solution
- Gurobi (MISOCP): 19 minutes to optimal

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Our new method (requires MILP solver and conic solver):

- CBC+ECOS: 5.5 hours to optimal
- Gurobi (MILP) + ECOS: 8 minutes to optimal

How general is MISOCP?

MINLPLIB2 benchmark library designed for derivative-based mixed-integer nonlinear solvers. We classified all of the convex instances by conic representability.

Of the 333 convex instances, **60%** are MISOCP representable.

- `tls6` instance previously unsolved. We translated it to Convex.jl and it was solved by Gurobi!

What about the remaining 40% of problems?

Of remaining 129 instances, in 107 the only nonlinear expressions involve exp and log.

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Let's consider the **exponential cone**:

$$EXP = \mathrm{cl}\{(x, y, z) \in \mathbb{R}^3 : y \exp(x/y) \leq z, y > 0\}$$

- Closed, convex, nonsymmetric cone. Theoretically tractable and supported by ECOS (Akle, 2015).

What about the remaining 22 of 333 instances?

- Seven representable by combination of *EXP* and second-order cones (*SOC*).
- Two representable by the power cone

$$POW_{\alpha} = \{(x, y, z) \in \mathbb{R}^3 : |z| \leq x^a y^{1-a}, x \geq 0, y \geq 0\}$$

- One family of 13 instances has a univariate convex function not known to be exactly representable by *SOC*, *EXP*, or *POW* (but could approximate).

Folklore: **Almost all convex optimization problems of practical interest can be represented as conic programming problems using second-order, positive semidefinite, exponential, and power cones.**

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Rest of this talk:

- How should we solve mixed-integer conic problems?
- What do we gain (if anything) from the translation to conic form?

$$\begin{aligned} \min_{x,z} \quad & c^T z \\ \text{s.t.} \quad & A_x x + A_z z = b \\ & L \le x \le U, x \in \mathbb{Z}^n, z \in \mathcal{K}, \end{aligned} \tag{MICONE}$$

- $\mathcal{K} = \mathcal{K}_1 \times \mathcal{K}_2 \times \cdots \times \mathcal{K}_r$, where each $\mathcal{K}_i$ is a standard cone: nonnegative orthant, second-order cone, exponential cone, power cone, or even positive semidefinite cone.
- WLOG, integer variables do not belong to cones, have zero objective coefficients

MILP solvers are powerful, can we use them to solve MICONE problems?

- To do so, need to approximate cone $\mathcal{K}$ with finite number of linear constraints (i.e., polyhedron)

Given a closed, convex cone $\mathcal{K}$, the **dual cone** $\mathcal{K}^*$ is the set such that $z \in \mathcal{K}$ iff $z^T\beta \geq 0 \ \forall\beta \in \mathcal{K}^*$.

- Nonnegative orthant, second-order and positive semidefinite cone are *self dual*. Exponential and power cones are not.
- Any finite subset of $\mathcal{K}^*$ yields a polyhedral outer approximation of $\mathcal{K}$!

$$\begin{aligned} \min_{x,z} \quad & c^T z \\ \text{s.t.} \quad & A_x x + A_z z = b \\ & L \le x \le U, x \in \mathbb{Z}^n, \\ & \beta^T z \ge 0 \, \forall \beta \in T, \end{aligned} \tag{MIOA(T)}$$

- If $T = \mathcal{K}^*$, then equivalent to MICONE
- If $T \subseteq \mathcal{K}^*$ and $|T| < \infty$, then *polyhedral outer approximation* $\rightarrow$ MILP relaxation of MICONE
- **Claim:** $\exists T \subseteq \mathcal{K}^*, |T| < \infty$ such that MIOA(T) is equivalent to *MICONE*

Consider the subproblem with integer values $x = \hat{x}$ fixed:

$$\begin{aligned} v_{\hat{x}} = \min_{z} \quad & c^T z \\ \text{s.t. } & A_z z = b - A_x \hat{x}, \\ & z \in \mathcal{K}. \end{aligned} \tag{$CP(\hat{x})$}$$

The dual of $CP(\hat{x})$ is

$$\begin{aligned} \max_{\beta,\lambda} \quad & \lambda^T (b - A_x \hat{x}) \\ \text{s.t. } & \beta = c - A_z^T \lambda \\ & \beta \in \mathcal{K}^*. \end{aligned}$$

- Start with $T = \emptyset$ (or small).
- Until the optimal objective of MIOA(T) is $\geq$ the optimal objective of the best feasible solution:
  - Let $\hat{x}$ be the optimal integer solution of the MILP problem MIOA(T)
  - Solve conic problem $CP(\hat{x})$, add dual solution $\beta_{\hat{x}}$ to T
  - If $CP(\hat{x})$ was feasible, record new feasible solution to MICONE

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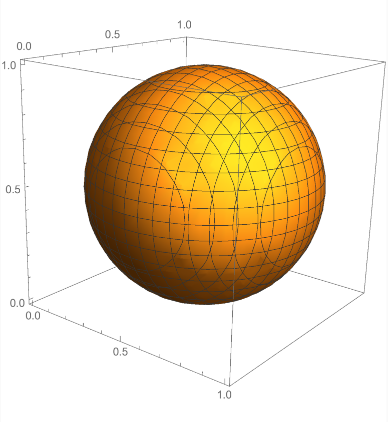
Finite termination assuming strong duality holds at subproblems

- Easy to implement, requires black box MILP and conic solvers
- Very often, number of iterations is < 10, sometimes 10-30. Worst observed is 171.
- Each MILP subproblem could be NP-Hard, but takes advantage of fast solvers

Hijazi et al. (2014):

$$B^n := \left\{ x \in \{0,1\}^n : \sum_{j=1}^{n} \left(x_j - \frac{1}{2} \right)^2 \leq \frac{n-1}{4} \right\}$$



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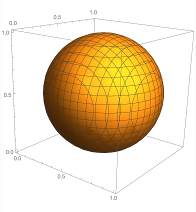

- B^n is empty
- No outer approximating hyperplane can exclude more than one integer lattice point
- So any polyhedral outer approximation which contains no integer vertices needs at least 2^n hyperplanes

$$B^n := \left\{ x \in \{0,1\}^n : \sum_{j=1}^{n} \left(x_j - \frac{1}{2} \right)^2 \leq \frac{n-1}{4} \right\}$$

Consider the equivalent formulation in an extended set of variables:

$$\hat{B}^n := \left\{ x \in \{0,1\}^n, z \in \mathbb{R}^n : \sum_{j=1}^{n} z_j \leq \frac{n-1}{4}, \left(x_j - \frac{1}{2} \right)^2 \leq z_j \; \forall j \right\}$$

- $B^n = \operatorname{proj}_x \hat{B}^n$

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- $B^n = \text{proj}_x \, \hat{B}^n$
- If you form a polyhedral relaxation of each $\left(x_j - \frac{1}{2}\right)^2 \leq z_j$ individually, then $2n$ hyperplanes in $\mathbb{R}^{2n}$ is sufficient to exclude all integer points

HOW TO FIX IT?

$$B^n := \left\{ x \in \{0,1\}^n : \sum_{j=1}^{n} \left(x_j - \frac{1}{2}\right)^2 \leq \frac{n-1}{4} \right\}$$

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- Outer approximation algorithm now converges in 2 iterations
- Why? **Small polyhedron in higher dimension can have exponentially many facets in lower dimension**

"EXTENDED FORMULATION FOR SECOND-ORDER CONE"

Vielma, Dunning, Huchette, Lubin (2015):

- For MISOCP: Rewrite
$$\sum_i x_i^2 \leq t^2$$
as
$$\sum_i z_i \leq t, \text{ where } z_i \geq x_i^2/t$$
- Implemented by CPLEX and Gurobi within months of publication.

Example:

$$\sum_i \exp(c_i^T x + b_i) \leq t$$

is expressed in conic form as

$$\sum_i z_i \leq t \text{ where } (c_i^T x + b_i, 1, z_i) \in EXP.$$

The translation to conic form already produces a formulation with additional variables ideal for polyhedral outer approximation.

Example:

$$\exp(x^2 + y^2) \leq t$$

is DCP compliant. CVX will generate equivalent formulation

$$\exp(z) \leq t \text{ where } x^2 + y^2 \leq z$$

which is then translated to EXP and second-order cones.

- Tawarmalani and Sahinidis (2005) show that polyhedral outer approximations that exploit composition structure have extra strength. This comes for “free” with conic representations.

$$
\begin{aligned}
v_{\hat{x}} = \min_{z} \quad & c^T z \\
\text{s.t. } & A_z z = b - A_x \hat{x}, \\
& z \in \mathcal{K}.
\end{aligned}
\qquad\qquad
\begin{aligned}
\max_{\beta, \lambda} \quad & \lambda^T (b - A_x \hat{x}) \\
\text{s.t. } & \beta = c - A_z^T \lambda \\
& \beta \in \mathcal{K}^*.
\end{aligned}
$$

Suppose $CP(\hat{x})$ is feasible and strong duality holds at the optimal primal-dual solution $(z_{\hat{x}}, \beta_{\hat{x}}, \lambda_{\hat{x}})$. Then for any z with $A_z z = b - A_x \hat{x}$ and $\beta_{\hat{x}}^T z \geq 0$, we have $c^T z \geq v_{\hat{x}}$.

$$v_{\hat{x}} = \min_{z} \quad c^T z \qquad\qquad \max_{\beta,\lambda} \quad \lambda^T(b - A_x\hat{x})$$
$$\text{s.t. } A_z z = b - A_x\hat{x}, \qquad\qquad \text{s.t. } \beta = c - A_z^T\lambda$$
$$z \in \mathcal{K}. \qquad\qquad \beta \in \mathcal{K}^*.$$

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Including $\beta_{\hat{x}}^T z \geq 0$ in polyhedral relaxation implies that if $x = \hat{x}$ is the optimal (integer part of the) solution of the relaxation, then the objective value of the relaxation is at least $v_{\hat{x}}$.

Proof.

$$\beta_{\hat{x}}^T z = (c - A_z^T\lambda_{\hat{x}})^T z = c^T z - \lambda_{\hat{x}}^T(b - A_x\hat{x}) = c^T z - v_{\hat{x}} \geq 0.$$

□

$$
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Given $\hat{x}$, assume $CP(\hat{x})$ is infeasible and its dual is unbounded, such that we have a ray $(\beta_{\hat{x}}, \lambda_{\hat{x}})$ satisfying $\beta_{\hat{x}} \in \mathcal{K}^*$, $\beta = -A_z^T \lambda_{\hat{x}}$, and $\lambda_{\hat{x}}^T (b - A_x \hat{x}) > 0$. Then for any z satisfying $A_z z = b - A_x \hat{x}$ we have $\beta_{\hat{x}}^T z < 0$.

Including $\beta_{\hat{x}}^T z \geq 0$ in polyhedral relaxation implies $x = \hat{x}$ is infeasible to the relaxation.

$$v_{\hat{x}} = \min_{z} \quad c^T z \qquad\qquad \max_{\beta,\lambda} \quad \lambda^T(b - A_x\hat{x})$$

$$\text{s.t. } A_z z = b - A_x\hat{x}, \qquad\qquad \text{s.t. } \beta = c - A_z^T\lambda$$

$$z \in \mathcal{K}. \qquad\qquad \beta \in \mathcal{K}^*.$$

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Including $\beta_{\hat{x}}^T z \geq 0$ in polyhedral relaxation implies $x = \hat{x}$ is infeasible to the relaxation.

Proof.

$$\beta_{\hat{x}}^T z = -\lambda_{\hat{x}}^T A_z z = -\lambda_{\hat{x}}^T(b - A_x\hat{x}) < 0.$$

□

Claim: $\exists T \subseteq \mathcal{K}^*, |T| < \infty$ such that MIOA(T) is equivalent to *MICONE*

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Proof.

Solve $CP(\hat{x})$ for all possible $\hat{x}$ (finite). Then include all corresponding dual vectors $\beta_{\hat{x}}$ in T. If x^* is the integer solution of MIOA(T), then $CP(x^*)$ is feasible from the second lemma. From first lemma, objective of MIOA(T) is at least the optimal value of $CP(x^*)$, but it's also a relaxation so x^* must be optimal. □

RECAP

- Value of disciplined modeling for mixed-integer convex:
    - Translation to conic form makes the problem easier to solve!
    - DCP rules correspond to extended formulations
- Almost all traditional benchmark problems are conic representable
- Proposed first outer approximation algorithm, based on conic duality

Computational results

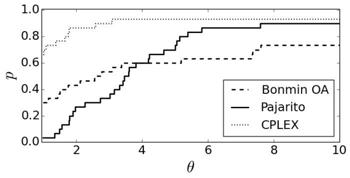
Pajarito

New solver!

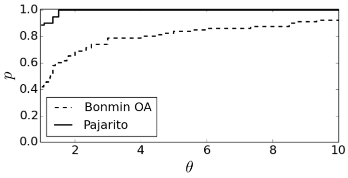
- 1000 lines of Julia
- Input is in conic form, accessible through Convex.jl DCP language
- Works with any MILP and conic solvers supported in Julia
- For tough conic problems, also supports traditional nonlinear solvers
- Open source pending DOE approval

Translated 194 convex instances from MINLPLIB2 from AMPL into Convex.jl (heroic parsing work by E. Yamangil).

Compare with Bonmin's outer approximation algorithm using CPLEX as the MILP solver. We use the same version of CPLEX plus KNITRO as NLP (conic) solver.

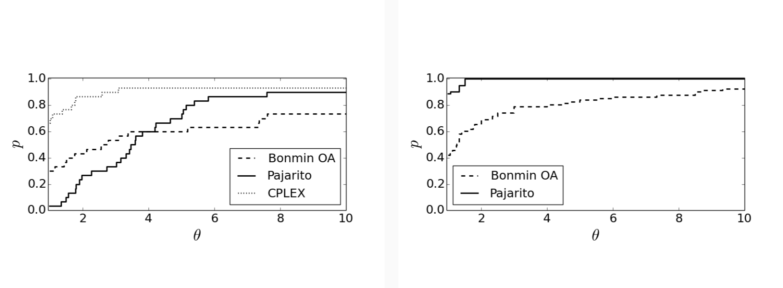


(a) Solution time



(b) Number of OA iterations

Figure: Comparison performance profiles for SOC representable instances (Bonmin >30 sec)

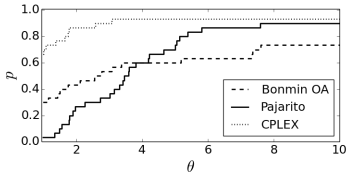


(a) Solution time

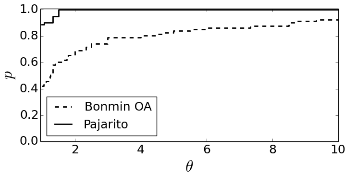
(b) Number of OA iterations

Figure: Comparison performance profiles for SOC representable instances (Bonmin >30 sec)

- Dominates on iteration count. Haven't optimized Pajarito so Bonmin is faster on the easier problems.



(a) Solution time



(b) Number of OA iterations

Figure: Comparison performance profiles for SOC representable instances (Bonmin >30 sec)

- Dominates on iteration count. Haven't optimized Pajarito so Bonmin is faster on the easier problems.
- CPLEX is usually the fastest on MISOCPs. Use that instead.

Almost none of the non-MISOCP instances in MINLPLIB2 are hard!

gams01

`gams01` unsolved instance which is EXP+SOC representable

- 145 variables. 1268 constraints (1158 are linear)
- Previous best bound: 1735.06. Best known solution: 21516.83
- Pajarito solved in 6 iterations ($<$ 10 hours). Optimal solution is 21380.20.

Thanks! Questions?

http://arxiv.org/abs/1511.06710

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